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Problem :- Every subspace of a T_0 - space is a T_0 - space that is to say, the property of a space being a T_0 - space is a hereditary property.

Sol :- Let $(X, \mathfrak{J})$ be a T_0 - space and (Y, U) a subspace of $(X, \mathfrak{J})$.

If we show tha (Y, U) is a T_0 - space, we conclude the result.

Let $x, y \in Y$ be arbitrary s.t. $y \neq x$

Then $x, y \in X$ s.t. $x \neq y$ for $Y \subset X$

By the property of a T_0 - space

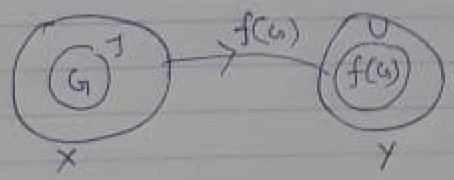
$\exists\ G \in \mathfrak{J}$ s.t. $x \in G$, $y \notin G$

Consequently $\exists\ G \cap Y \in U$ s.t. $x \in G \cap Y$, $y \notin G \cap Y$

$\Rightarrow (Y, U)$ is a T_0 - space.

Th.B :- The property of a space being a T_1 - space is a topological property i.e. the homeomorphic image of a T_1 - space is a T_1 - space.

Proof :- Let $(X, \mathfrak{J})$ be a T_1 - space and let $f : (X, \mathfrak{J}) \to (Y, U)$ be a homeomorphic and so $G \in \mathfrak{J} \Rightarrow f(G) \in U$



To prove that T_1 - space is a topological property, it is enough to prove that (Y, U) is a T_1 - space.

$\because$ X is a T_1 - space

given a pair of distinct elements $p, q \in X$;

$\exists\ G, H \in \mathfrak{J}$ s.t. $p \in G$, $q \notin G$; $q \in H$, $p \notin H$

Therefore given $f(p), f(q) \in Y$ s.t.
$f(p) \neq f(q)$

$\exists\, f(G), f(H) \in U$ s.t.
$f(p) \in f(G)$, $f(q) \notin f(G)$

and $f(q) \in f(H)$, $f(p) \notin f(H)$
This proves that (Y, U) is a T_1-space

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(27) Th. 17 :- In a T_2 - Space, a convergent sequence has a unique limit.

Proof :- Let $(X, \mathcal{T})$ be a H- Space. Let $\langle a_n \rangle$ be a convergent sequence in X

To prove that the sequence $\langle a_n \rangle$ has a unique limit

Suppose contradiction

Then $\langle a_n \rangle$ cannot have a unique limit.

Let $\langle a_n \rangle$ converges to distinct points, let $a_0, b_0 \in X$

Then $a_0 \neq b_0$

by def of convergence

$a_0 \in G \in \mathcal{T} \Rightarrow \exists\, n_0 \in N$ s.t. $\forall\, n \geq n_0 \Rightarrow a_n \in G$

and $b_0 \in H \in \mathcal{T} \Rightarrow \exists\, k_0 \in N$ s.t. $\forall\, n \geq k_0 \Rightarrow a_n \in H$

let $m_0 = \max(n_0, k_0)$

Then $\forall\, n \geq m_0 \Rightarrow a_n \in G, a_n \in H \Rightarrow a_n \in G \cap H$

$\Rightarrow G \cap H \neq \phi$

given a pair of distinct elements $a_0, b_0 \in X$ there are open sets, $G, H \subset X$ s.t. $a_0 \in G$, $b_0 \in H$, $G \cap H \neq \phi$

which proves that $(X, \mathcal{T})$ is not a H- space.

Hence $\langle a_n \rangle$ has a unique limit.